2.3 DERIVADA DE UNA FUNCIÓN COMPUESTA. REGLA DE LA CADENA

$$F(x) = (g\ f)(x) = g(f(x)) \quad \rightarrow \quad F'(x) = g'(f(x)) \cdot f'(x)$$

Las reglas de derivación aplicadas a funciones compuestas quedan así:

- $F(x) = (f(x))^n \quad \rightarrow \quad F'(x) = n \cdot (f(x))^{n-1} \cdot f'(x)$
- $F(x) = sen\ (f(x)) \quad \rightarrow \quad F'(x) = cos\ (f(x)) \cdot f'(x)$
- $F(x) = cos\ (f(x)) \quad \rightarrow \quad F'(x) = -sen\ (f(x)) \cdot f'(x)$
- $F(x) = tg\ (f(x)) \quad \rightarrow \quad F'(x) = (1 + tg^2\ (f(x)) \cdot f'(x) = \dfrac{f'(x)}{cos^2\ (f(x))}$
- $F(x) = arc\,sen\ (f(x)) \quad \rightarrow \quad F'(x) = \dfrac{f'(x)}{\sqrt{1 - (f(x))^2}}$
- $F(x) = arc\,cos\ (f(x)) \quad \rightarrow \quad F'(x) = \dfrac{-f'(x)}{\sqrt{1 - (f(x))^2}}$
- $F(x) = arc\,tg\ (f(x)) \quad \rightarrow \quad F'(x) = \dfrac{f'(x)}{1 + (f(x))^2}$
- $F(x) = e^{f(x)} \quad \rightarrow \quad F'(x) = e^{f(x)} \cdot f'(x)$
- $F(x) = a^{f(x)} \quad \rightarrow \quad F'(x) = a^{f(x)} \cdot f'(x) \cdot ln\ a$
- $F(x) = ln\ (f(x)) \quad \rightarrow \quad F'(x) = \dfrac{f'(x)}{f(x)}$
- $F(x) = log_a\ (f(x)) \quad \rightarrow \quad F'(x) = \dfrac{f'(x)}{f(x)} \cdot \dfrac{1}{ln\ a}$

EJERCICIO RESUELTO

Halla la derivada de estas funciones:

a) $f(x) = (x^2 - 3x)^7$

b) $g(x) = sen^3\ x = (sen\ x)^3$

c) $b(x) = ln^3\ (x^2 + 3) = [ln\ (x^2 + 3)]^3$

RESOLUCIÓN

a) $f'(x) = 7(x^2 - 3x)^6 \cdot (2x - 3) = (14x - 21)\ (x^2 - 3x)^6$

b) $g'(x) = 3\ sen^2\ x \cdot cos\ x$

c) $b'(x) = 3\ [ln\ (x^2 + 3)]^2 \cdot \dfrac{1}{x^2 + 3} \cdot 2x = \dfrac{6x\ ln^2\ (x^2 + 3)}{x^2 + 3}$

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Halla la derivada de las siguientes funciones:

1. $f(x) = (x^2 + 5)^6 \rightarrow f'(x) =$
2. $f(x) = sen\,(x^2 - 1) \rightarrow f'(x) =$
3. $f(x) = cos\,(ln\,x) \rightarrow f'(x) =$
4. $f(x) = tg\,(2x - 3x^2) \rightarrow f'(x) =$
5. $f(x) = e^{3x^2 + 1} \rightarrow f'(x) =$
6. $f(x) = 2^{4x + 1} \rightarrow f'(x) =$
7. $f(x) = cos^2 x \rightarrow f'(x) =$
8. $f(x) = e^{3x} \rightarrow f'(x) =$
9. $f(x) = ln\,(3x^2 - 6) \rightarrow f'(x) =$
10. $f(x) = ln\left(\frac{3x^2 - 1}{2}\right) \rightarrow f'(x) =$
11. $f(x) = arc\,tg\,(3x^2 + 2x) \rightarrow f'(x) =$
12. $f(x) = arc\,sen\,(x^2) \rightarrow f'(x) =$
13. $f(x) = arc\,cos\,(x^3 - 1) \rightarrow f'(x) =$
14. $f(x) = sen\,(3x^2 - 1)^2 \rightarrow f'(x) =$
15. $f(x) = sen^2\,(3x^2 - 1) \rightarrow f'(x) =$
16. $f(x) = 3^{cos\,x} \rightarrow f'(x) =$
17. $f(x) = ln\left(\frac{x + 1}{x - 2}\right) \rightarrow f'(x) =$
18. $f(x) = \left(\frac{x^2 - 1}{x + 2}\right)^2 \rightarrow f'(x) =$
19. $f(x) = \sqrt{x^2 - 4x} \rightarrow f'(x) =$
20. $f(x) = \frac{x + 1}{(x - 2)^2} \rightarrow f'(x) =$
21. $f(x) = \frac{(2x + 1)^2}{x - 1} \rightarrow f'(x) =$
22. $f(x) = \frac{(3x - 1)^2}{2x + 1} \rightarrow f'(x) =$
23. $f(x) = \frac{e^x}{(x - 1)^2} \rightarrow f'(x) =$

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$f'(x) = \frac{-2x}{(x^2+1)^2}$

$f'(x) = \frac{3x^2(x+2) - x^3 \cdot 1}{(x+2)^2} = \frac{2x^3 + 6x^2}{(x+2)^2}$

$f'(x) = \frac{2(3x+2) - (2x-1)\cdot 3}{(3x+2)^2} = \frac{7}{(3x+2)^2}$

$f'(x) = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$

$f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+2) - \sqrt{x}\cdot 1}{(x+2)^2} = \frac{-x+2}{2\sqrt{x}\cdot(x+2)^2}$

$f'(x) = 2x \cdot \sqrt{x} + (x^2-1)\cdot\frac{1}{2\sqrt{x}} =$

$= \frac{4x^2 + x^2 - 1}{2\sqrt{x}} = \frac{5x^2-1}{2\sqrt{x}}$

$f'(x) = \frac{3}{\sqrt{1-x^2}}$

$f'(x) = \frac{-2}{\sqrt{1-x^2}} + e^x$

$f'(x) = \frac{5}{1+x^2}$

$f'(x) = \frac{1\cdot e^x + x\,e^x - \frac{1}{x}}{2} = \frac{x\,e^x + x^2\,e^x - 1}{2x}$

$f'(x) = 3^x \cdot \ln 3 \cdot \operatorname{sen} x + 3^x \cdot \cos x - \left(\log_2 x + \right.$

$\left. + x \cdot \frac{1}{x} \cdot \frac{1}{\ln 2}\right) = 3^x \ln 3 \cdot \operatorname{sen} x +$

$+ 3^x \cos x - \log_2 x - \frac{1}{\ln 2}$

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1. $f'(x) = 6(x^2+5)^5(2x) = 12x(x^2+5)^5$
2. $f'(x) = \cos(x^2-1)\cdot 2x = 2x\cos(x^2-1)$
3. $f'(x) = -\operatorname{sen}(\ln x)\cdot\frac{1}{x} = \frac{-1}{x}\operatorname{sen}(\ln x)$
4. $f'(x) = [1 + tg^2(2x-3x^2)]\cdot(2-6x)$
5. $f'(x) = e^{3x^2+1}\cdot 6x = 6x\cdot e^{3x^2+1}$
6. $f'(x) = 2^{4x+1}\cdot\ln 2\cdot 4 = (4\cdot\ln 2)\cdot 2^{4x+1}$
7. $f'(x) = 2\cos x(-\operatorname{sen} x) = -2\cos x\ \operatorname{sen} x$
8. $f'(x) = e^{3x}\cdot 3 = 3\cdot e^{3x}$
9. $f'(x) = \frac{6x}{3x^2-6}$

$f'(x) = \frac{3x}{\frac{3x^2-1}{2}} = \frac{6x}{3x^2-1}$

$f'(x) = \frac{6x+2}{1+(3x^2+2x)^2} = \frac{6x+2}{9x^4+12x^3+4x^2+1}$

$f'(x) = \frac{2x}{\sqrt{1-x^4}}$

$f'(x) = \frac{-3x^2}{\sqrt{1-(x^3-1)^2}} = \frac{-3x^2}{\sqrt{2x^3-x^6}} =$

$= \frac{-3x^2}{x\sqrt{2x-x^4}} = \frac{-3x}{\sqrt{2x-x^4}}$

$f'(x) = (\cos(3x^2-1)^2)\cdot 2(3x^2-1)\cdot 6x =$

$= 12x(3x^2-1)\cos(3x^2-1)^2$

$f'(x) = 2\operatorname{sen}(3x^2-1)\cdot\cos(3x^2-1)\cdot 6x =$

$= 12x\operatorname{sen}(3x^2-1)\cos(3x^2-1)$

$f'(x) = 3^{\cos x}\cdot\ln 3\cdot(-\operatorname{sen} x) = -\operatorname{sen} x\cdot\ln 3\cdot 3^{\cos x}$

$f'(x) = \frac{1}{\frac{x+1}{x-2}}\cdot\frac{1\cdot(x-2)-(x+1)\cdot 1}{(x-2)^2} =$

$= \frac{(x-2)}{(x+1)}\cdot\frac{-3}{(x-2)^2} = \frac{-3}{(x+1)(x-2)} =$

$= \frac{-3}{x^2-x-2}$

$f'(x) = 2\left(\frac{x^2-1}{x+2}\right)\cdot\frac{2x(x+2)-(x^2-1)\cdot 1}{(x+2)^2} =$

$= \frac{2(x^2-1)(x^2+4x+1)}{(x+2)^3}$

$f'(x) = \frac{2x-4}{2\sqrt{x^2-4x}} = \frac{x-2}{\sqrt{x^2-4x}}$

$f'(x) = \frac{1\cdot(x-2)^2 - (x+1)\cdot 2(x-2)}{(x-2)^4} =$

$= \frac{(x-2)[1-2(x+1)]}{(x-2)^4} = \frac{2x-1}{(x-2)^3}$

$f'(x) = \frac{2(2x+1)\cdot 2(x-1) - (2x+1)^2\cdot 1}{(x-1)^2} =$

$= \frac{(2x+1)(2x-5)}{(x-1)^2} = \frac{4x^2-8x-5}{(x-1)^2}$

$f'(x) = \frac{2(3x-1)\cdot 3(2x+1) - (3x-1)^2\cdot 2}{(2x+1)^2} =$

$= \frac{(3x-1)(6x+8)}{(2x+1)^2} = \frac{18x^2+18x-8}{(2x+1)^2}$

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