

2.1 DERIVADA DE UNA POTENCIA, DE UNA SUMA Y DEL PRODUCTO POR UN NÚMERO

- $f(x) = k$ (constante) $\rightarrow f'(x) = 0$
- $f(x) = x$ $\rightarrow f'(x) = 1$
- $f(x) = x^n$ $\rightarrow f'(x) = nx^{n-1}$
- $F(x) = f(x) \pm g(x)$ $\rightarrow F'(x) = f'(x) \pm g'(x)$
- $F(x) = k \cdot f(x)$ $\rightarrow F'(x) = k \cdot f'(x)$

EJERCICIO RESUELTO

Halla la derivada de las siguientes funciones:

a) $f(x) = x^4 - \frac{3}{4}x^3 + 2x - 1$ b) $f(x) = \sqrt{x}$

c) $f(x) = \frac{3}{5x^4}$ d) $f(x) = \frac{x^2}{\sqrt[3]{x}}$

RESOLUCIÓN

a) $f'(x) = 4x^3 - \frac{3}{4} \cdot 3x^2 + 2 = 4x^3 - \frac{9}{4}x^2 + 2$

b) $f(x) = x^{1/2} \rightarrow f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

c) $f(x) = \frac{3}{5}x^{-4} \rightarrow f'(x) = \frac{3}{5} \cdot (-4) \cdot x^{-4-1} = \frac{-12}{5}x^{-5} = \frac{-12}{5x^5}$

d) $f(x) = \frac{x^2}{x^{1/3}} = x^{2-1/3} = x^{5/3} \rightarrow f'(x) = \frac{5}{3}x^{2/3} = \frac{5\sqrt[3]{x^2}}{3}$

Halla la derivada de cada una de estas funciones:

1 $f(x) = 2x + 1 \rightarrow f'(x) =$

2 $f(x) = \frac{3x-2}{4} \rightarrow f'(x) =$

3 $f(x) = \frac{3}{4} \rightarrow f'(x) =$

4 $f(x) = \frac{x}{2} + 3 \rightarrow f'(x) =$

5 $f(x) = x^3 - 3x^2 + 2 \rightarrow f'(x) =$

6 $f(x) = \frac{3x^5}{5} - \frac{4x}{3} + 5 \rightarrow f'(x) =$

7 $f(x) = \frac{4\pi - 2}{3} \rightarrow f'(x) =$

$$\diamond 8 \quad f(x) = \frac{4}{3}(x^2 - \frac{3}{4}x + 2) \quad \rightarrow \quad f'(x) =$$

$$\diamond 9 \quad f(x) = \frac{x^2}{5} - \frac{x}{4} + \sqrt{5} \quad \rightarrow \quad f'(x) =$$

$$\diamond 10 \quad f(x) = \frac{x}{7} - \sqrt{7x} = \frac{x}{7} - \sqrt{7} \cdot \sqrt{x} \quad \rightarrow \quad f'(x) =$$

$$\diamond 11 \quad f(x) = \frac{1}{x} \quad \rightarrow \quad f'(x) =$$

$$\diamond 12 \quad f(x) = \frac{3}{x^2} \quad \rightarrow \quad f'(x) =$$

$$\diamond 13 \quad f(x) = \frac{5}{3x^3} \quad \rightarrow \quad f'(x) =$$

$$\diamond 14 \quad f(x) = \sqrt[5]{x^4} \quad \rightarrow \quad f'(x) =$$

$$\diamond 15 \quad f(x) = \frac{\sqrt{3x}}{x^2} \quad \rightarrow \quad f'(x) =$$

$$\diamond 16 \quad f(x) = \frac{3\sqrt{x^3}}{2x^4} \quad \rightarrow \quad f'(x) =$$

$$\diamond 17 \quad f(x) = \frac{2}{x} + \frac{x}{2} \quad \rightarrow \quad f'(x) =$$

$$\diamond 18 \quad f(x) = \frac{\sqrt[4]{x^2}}{3} - \frac{x}{3} + \sqrt{5} \quad \rightarrow \quad f'(x) =$$

$$\diamond 19 \quad f(x) = \sqrt[4]{\frac{1}{x^3}} \quad \rightarrow \quad f'(x) =$$

$$\diamond 20 \quad f(x) = \sqrt{\frac{3}{x^5}} \quad \rightarrow \quad f'(x) =$$

$$\diamond 21 \quad f(x) = \frac{2\sqrt{x}}{x} - \frac{3}{x^2} + \frac{1}{x} \quad \rightarrow \quad f'(x) =$$

$$\diamond 22 \quad f(x) = x - \frac{3\sqrt{5}}{4} + \frac{1}{x^2} \quad \rightarrow \quad f'(x) =$$

$$\diamond 23 \quad f(x) = \frac{x^2}{3} - \frac{3}{x^2} + \frac{3\sqrt{5}}{2} \quad \rightarrow \quad f'(x) =$$

$$\diamond 24 \quad f(x) = \frac{x^3}{3} - 4\sqrt{x} - \frac{2}{x^3} - x^2\sqrt{x} \quad \rightarrow \quad f'(x) =$$

$$\diamond 25 \quad f(x) = \frac{x^2 - 3x + 1}{x} = \frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x} \quad \rightarrow \quad f'(x) =$$

b) $f'(2) = 0$ (la recta tangente es horizontal, luego tiene pendiente 0).

c) $f'(4)$ es la pendiente de la recta que pasa por $(3, 3)$ y $(4, 0)$:

$$f'(4) = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$$\begin{aligned} \diamond f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h} = \\ &= \lim_{h \rightarrow 0} \frac{a^2 + h^2 + 2ah - a^2}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 2ah}{h} = \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{h(h+2a)}{h} = \lim_{h \rightarrow 0} (h+2a) = 2a$$

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- \diamond a) $f'(x) = 3$ b) $f'(x) = \frac{1}{5}$
 c) $f'(x) = \frac{-1}{2}$ d) $f'(x) = 1$
 e) $f'(x) = 2x$ f) $f'(x) = 0$
 g) $f'(x) = 0$ h) $f'(x) = 4x - 1$
 i) $f'(x) = \frac{-1}{x^2}$ j) $f'(x) = \frac{-1}{(x+1)^2}$
 k) $f'(x) = \frac{-6}{(3x-1)^2}$

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- \diamond $f'(x) = 2$
 \diamond $f'(x) = \frac{3}{4}$
 \diamond $f'(x) = 0$
 \diamond $f'(x) = \frac{1}{2}$
 \diamond $f'(x) = 3x^2 - 6x$
 \diamond $f'(x) = 3x^4 - \frac{4}{3}$
 \diamond $f'(x) = 0$

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$$\diamond f'(x) = \frac{4}{3} \left(2x - \frac{4}{3} \right)$$

$$\diamond f'(x) = \frac{2x}{5} - \frac{1}{4}$$

$$\diamond f'(x) = \frac{1}{7} - \frac{\sqrt{7}}{2\sqrt{x}}$$

$$\diamond f'(x) = \frac{-1}{x^2}$$

$$\diamond f'(x) = \frac{-6}{x^3}$$

$$\diamond f'(x) = \frac{-5}{x^4}$$

$$\diamond f'(x) = \frac{4}{3} \sqrt[3]{x}$$

$$\begin{aligned} \diamond f'(x) &= \sqrt{3} \cdot x^{-3/2} \rightarrow f'(x) = \frac{-3\sqrt{3}}{2} \cdot \frac{1}{\sqrt{x^5}} = \\ &= \frac{-3\sqrt{3}}{2\sqrt{x^5}} \end{aligned}$$

$$\diamond f'(x) = \frac{3}{2} x^{-5/2} \rightarrow f'(x) = \frac{-15}{4\sqrt{x^7}}$$

$$\diamond f'(x) = \frac{-2}{x^2} + \frac{1}{2}$$

$$\diamond f'(x) = \frac{1}{3\sqrt{x}} - \frac{1}{3}$$

$$\diamond f'(x) = x^{-3/4} \rightarrow f'(x) = \frac{-3}{4\sqrt{x^7}}$$

$$\diamond f'(x) = \frac{\sqrt{3}}{\sqrt{x^5}} = \sqrt{3} \cdot x^{-5/2} \rightarrow f'(x) = \frac{-5\sqrt{3}}{2\sqrt{x^7}}$$

$$\diamond f'(x) = 2x^{-1/2} - 3x^{-2} + x^{-1} \rightarrow$$

$$\rightarrow f'(x) = \frac{-1}{\sqrt{x^3}} + \frac{6}{x^3} - \frac{1}{x^2}$$

$$\diamond f'(x) = 1 - \frac{2}{x^3}$$

$$\diamond f'(x) = \frac{2x}{3} + \frac{6}{x^3}$$

$$\diamond f'(x) = \frac{x^3}{3} - 4x^{1/2} + 2x^{-3} + x^{2/5} \rightarrow$$

$$\rightarrow f'(x) = x^2 - \frac{2}{\sqrt{x}} - \frac{6}{x^4} + \frac{5\sqrt{x^3}}{2}$$

$$\diamond f'(x) = 1 - \frac{1}{x^2}$$

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$$\diamond f'(x) = 3 \cos x + 2 \operatorname{sen} x$$