

POTENCIA DE UNA MATRIZ

Ejercicios

1. Dada la matriz $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, determina A^{2015}
2. Dada la matriz $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Calcula A^{127} y A^{128}
3. Dada la matriz $A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix}$. Calcula razonadamente A^{100}
4. Dada la matriz $A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix}$. Calcula A^{10} .

SOLUCIONES

1. Dada la matriz $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, determina A^{2015}

$$A^2 = A \cdot A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Luego: } A^{2015} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2. Dada la matriz $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Calcula A^{127} y A^{128}

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^2 = A \cdot A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$A^3 = A^2 \cdot A = I \cdot A = A$$

Por inducción vemos que:

$$A^{127} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = A$$

$$A^{128} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

3. Dada la matriz $A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix}$. Calcula razonadamente A^{100}

$$A^2 = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 4 & 4 \\ -1 & -3 & -3 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 1 \\ 1 & 4 & 4 \\ -1 & -3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Luego se cumple que $A^3 = -I$

$$A^{100} = A^{99} \cdot A = (A^3)^{33} \cdot A = (-I)^{33} \cdot A = -A = \begin{pmatrix} 0 & -3 & -4 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$$

4. Dada la matriz $A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix}$. Calcula A^{10} .

$$A^2 = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 4 & 4 \\ -1 & -3 & -3 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 3 & 4 \\ 1 & -4 & -5 \\ -1 & 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 1 \\ 1 & 4 & 4 \\ -1 & -3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$A^{10} = A^3 \cdot A^3 \cdot A^3 \cdot A = (-I) \cdot (-I) \cdot (-I) \cdot A = -A = \begin{pmatrix} 0 & -3 & -4 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$$